Based upon Dan Simon's Mathematica Kalman filtering example calculation.

Kalman Filter Example Calculation

Delta t: $\Delta t := 0.5$ Iterations: $n := 60$

Acceleration noise: $a_{noise} := 0.5$

Position Measurement noise: $PM_{noise} := 10$

Transition Matrix: $a := \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}$

Measurement Matrix: $c := \begin{pmatrix} 1 & 0 \end{pmatrix}$

Initial State Vector: $x_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Initial State Estimate: $x_{hat_0} := x_0$

Process noise covariance: $Q := a_{noise}^{2} \cdot \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 \end{bmatrix}$

Initial estimation covariance: $P_0 := Q$

Measurement error covariance: $R := PM_{noise}^{2}$

Define a matrix of random numbers with mean 0 and $\sigma = a_{noise}$: $randn_{process} := rnorm(n + 1, 0, a_{noise})$

Define a matrix of random numbers with mean 0 and $\sigma = PM_{noise}$: $randn_{measurement} := rnorm(n + 1, 0, PM_{noise})$

Simulate the process: $x_{j+1} := a \cdot x_j + \text{randn}_{\text{process}_j} \cdot \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}$

Simulate the measurement: $z_j := c \cdot x_j + \text{randn}_{\text{measurement}_j}$

Calculate the estimated position:

$$
x_{hat} := \left|
\begin{array}{l}
x_{hat_0} \leftarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
P_0 \leftarrow Q \\
\text{for } j \in 0..n \\
\quad \left|
\begin{array}{l}
s_j \leftarrow c \cdot P_j \cdot c^T + R \\
K_j \leftarrow \left(a \cdot P_j \cdot c^T\right) \cdot \left(s_j\right)^{-1} \\
I_j \leftarrow z_j - c \cdot \left(a \cdot x_{hat_j}\right) \\
P_{j+1} \leftarrow a \cdot P_j \cdot a^T + Q - K_j \cdot c \cdot P_j \cdot a^T \\
x_{hat_{j+1}} \leftarrow a \cdot x_{hat_j} + K_j \cdot I_j
\end{array}
\right. \\
x_{hat}
\end{array}
\right.
$$

Covariance of innovation: $s_j \leftarrow c \cdot P_j \cdot c^T + R$

Gain matrix: $K_j \leftarrow \left(a \cdot P_j \cdot c^T\right) \cdot \left(s_j\right)^{-1}$

Innovation: $I_j \leftarrow z_j - c \cdot \left(a \cdot x_{hat_j}\right)$

Covariance of prediction error: $P_{j+1} \leftarrow a \cdot P_j \cdot a^T + Q - K_j \cdot c \cdot P_j \cdot a^T$

Estimated position: $x_{hat_{j+1}} \leftarrow a \cdot x_{hat_j} + K_j \cdot I_j$

State estimate: